Example Candidate Responses

Cambridge International AS & A Level

Cambridge International AS and A Level Mathematics

9709

Paper 5

Cambridge Advanced

CAMBRIDGE
International Examinations

Version 1.0

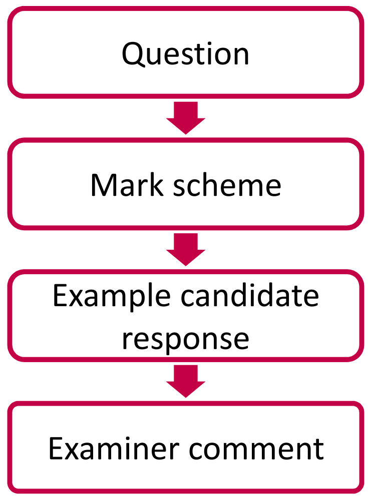
Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics (9709), and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen to exemplify a range of answers. Each response is accompanied by a brief commentary explaining the strengths and weaknesses of the answers.

For ease of reference the following format for each component has been adopted:



Each question is followed by an extract of the mark scheme used by examiners. This, in turn, is followed by examples of marked candidate responses, each with an examiner comment on performance. Comments are given to indicate where and why marks were awarded, and how additional marks could have been obtained. In this way, it is possible to understand what candidates have done to gain their marks and what they still have to do to improve them.

Past papers, Examiner Reports and other teacher support materials are available on Teacher Support at https://teachers.cie.org.uk

Assessment at a glance

The 7 units in the scheme cover the following subject areas:

- Pure Mathematics (units P1, P2 and P3);
- Mechanics (units M1 and M2);
- Probability and Statistics (units S1 and S2).

Centres and candidates may:

- take all four Advanced (A) Level components in the same examination session for the full A Level.
- follow a staged assessment route to the A Level by taking two Advanced Subsidiary (AS) papers (P1 & M1 **or** P1 & S1) in an earlier examination session;
- take the Advanced Subsidiary (AS) qualification only.

AS Level candidates take:

Paper 1: Pure Mathematics 1 (P1)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 60% of total

plus **one** of the following papers:

Paper 2: Pure Mathematics 2 (P2)	Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics (S1)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total

A Level candidates take:

Paper 1: Pure Mathematics 1 (P1)	Paper 3 Pure Mathematics 3 (P3)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 30% of total	**1¾ hours** About 10 shorter and longer questions 75 marks weighted at 30% of total

plus **one** of the following combinations of two papers:

Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics 1 (S1)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

Paper 4: Mechanics 1 (M1)	Paper 5: Mechanics 2 (M2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

Paper 6: Probability and Statistics 1 (S1)	Paper 7: Probability and Statistics 2 (S2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

Teachers are reminded that the latest syllabus is available on our public website at **www.cie.org.uk** and Teacher Support at **https://teachers.cie.org.uk**

Paper 5

Question 1

1 ABC is a uniform semicircular arc with diameter $AC = 0.5$ m. The arc rotates about a fixed axis through A and C with angular speed 2.4 rad s^{-1}. Calculate the speed of the centre of mass of the arc. [3]

Mark scheme

1	$OG = 0.25 \sin(\pi/2)/(\pi/2)$	B1	0.159 (15..)
	$v = 0.159 \times 2.4$	M1	
	$v = 0.382 \text{ ms}^{-1}$	A1√ [3]	√ 2.4 × cv (OG)

Example candidate response – 1

1

$$G = \frac{2r\sin\theta}{3\theta} \qquad r = 0.5/2 \qquad r = 0.25$$

$$= \frac{2(0.25)\sin\left(\frac{\pi}{2}\right)}{3\left(\frac{\pi}{2}\right)} = 0.1061$$

D0

$V = \omega r$

M1

$\cancel{\omega = \frac{v}{r}}$ $\qquad V = 2.4 \times 0.1061$

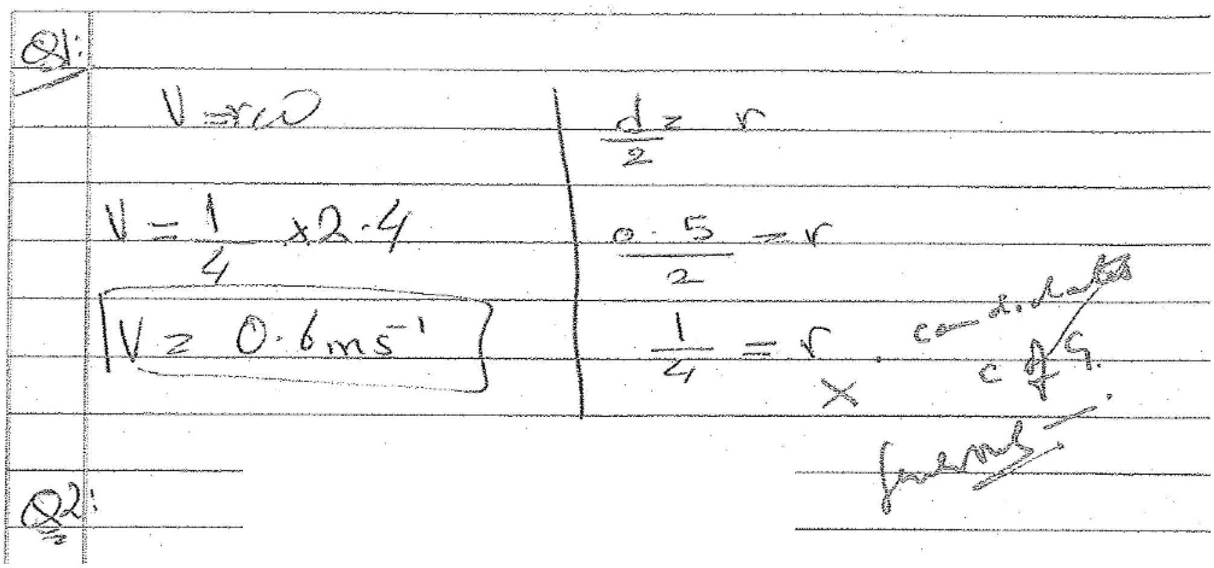
A1

$V = \boxed{0.255\ ms^{-1}}$

2

Total mark awarded = 2 out of 3

Example candidate response – 2



Q1:

$V = r\omega$

$V = \frac{1}{4} \times 2.4$

$\boxed{V = 0.6\,\text{ms}^{-1}}$

$\frac{d}{2} = r$

$\frac{0.5}{2} = r$

$\frac{1}{4} = r$

Q2:

Total mark awarded = 1 out of 3

Examiner comment – 1 and 2

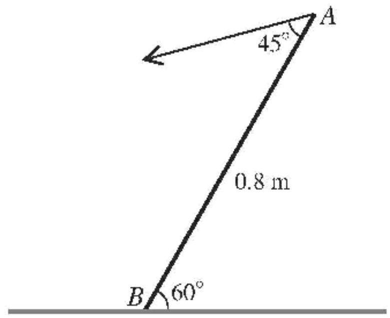
Candidate 1 made the mistake of using the incorrect formula for the centre of mass of the semi-circular lamina. $2r \sin \alpha/3\alpha$ was used instead of $r \sin \alpha/\alpha$. This value was then substituted into $v = r\omega$.

Candidate 2 did not attempt to find the centre of mass of the semi-circular lamina. $r = \frac{1}{4}$, the radius of the semi-circle, and $\omega = 2.4$ were substituted into $v = r\omega$ in order to find the speed.

The correct centre of mass = 0.25 $\sin(\pi/2)/(\pi/2)$ = 0.159

Question 2

2



A uniform rod AB has weight 6 N and length 0.8 m. The rod rests in limiting equilibrium with B in contact with a rough horizontal surface and AB inclined at 60° to the horizontal. Equilibrium is maintained by a force, in the vertical plane containing AB, acting at A at an angle of 45° to AB (see diagram). Calculate

(i) the magnitude of the force applied at A, [3]

(ii) the least possible value of the coefficient of friction at B. [4]

Mark scheme

2 (i)		M1	Takes moments about B
	$6 \times 0.4\cos 60 = 0.8\,P\cos 45$	A1	P is the force at A
	$P = 2.12\text{N}$	A1 [3]	
(ii)	$F = P\sin 75$ (F is friction force at B)	B1	Must use correct angle (cos15)
	$R = 6 + P\cos 75$ (R is normal reaction at B)	B1	Must use correct angle (sin15)
	$\mu = (2.12\sin 75)/(6 + 2.12\cos 75)$	M1	
	$\mu = 0.313$	A1 [4]	

Example candidate response – 1

(2)

i) $0.8F\sin 45 = 0.4(6)\sin 30$

$F = \frac{3\sqrt{2}}{2} \simeq 2.12$ N

3.

ii) $R = 6\cos 30 + 2.12\cos 45 = 6.696$ B0

$2.12\sin 75 = 6.696\mu$ B1 M1

$\mu = 0.306$ A0

(5)

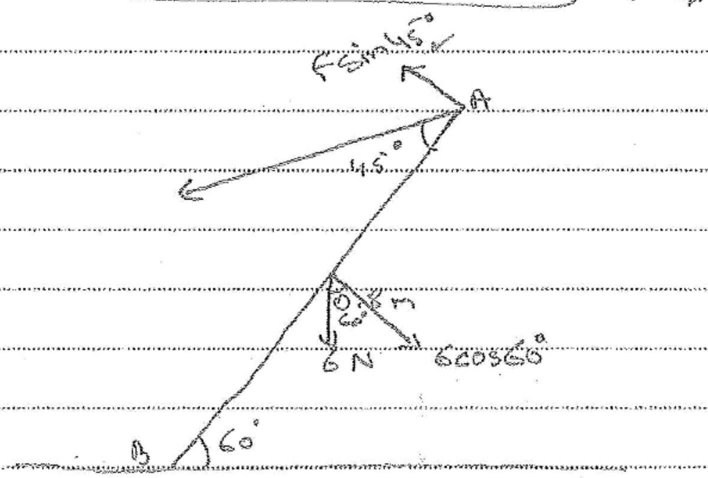
Item marks awarded: (i) = 3/3; (ii) = 2/4

Total mark awarded = 5 out of 7

Example candidate response – 2

$\boxed{\omega = 3.25 \text{ rad s}^{-1}}$ Ans//

Q.2.



(i)

taking moments about B,

$F \sin 45° \times 0.8 = 6 \cos 60° \times 0.4$

$\frac{2\sqrt{2}}{5} F = \frac{6}{5}$

$F = \frac{6^{3}}{12\sqrt{2}}$

$\cancel{F =}$

$F = 2.12 \text{ N}$

f at A = $(2.12)(\cos 45°)$

3.

$\boxed{= 3 \text{ N}}$ Ans//

Example candidate response – 2, continued

(ii)

$R = 6 + (3\cos 45^\circ)\cos 30^\circ$

$\boxed{R = 7.84\text{ N}}$

$F = 3 + 6$

$= 9\text{ N}$

$F = \mu R$

$\mu =$

$R = 6\text{ N}$

$F = 9\text{ N}$

$F = \mu R$

$9 = \mu 6$

$\mu =$

$R = 7.84\text{ N}$ ✗ B0

$F = 9\text{ N}$ B0

$F = \mu R$ M1

$\mu = \dfrac{9}{7.84}$ A0

$\boxed{= 1.15}$ Ans ✗

4

Item marks awarded: (i) = 3/3; (ii) = 1/4

Total mark awarded = 4 out of 7

Examiner comment – 1 and 2

(i) Both candidates scored all three marks for this part, using suitable methods. The correct moment equation about B was seen. This equation was $0.8F \sin 45 = 0.4 \times 6 \sin 30$ or $0.4 \times 6 \cos 60$, and led to $F = 2.12\,\text{N}$.

(ii) Candidate 1 found the friction force correctly. An error was made in finding the normal reaction, R. $F = \mu R$ was then applied. By resolving vertically, $R = 6 + 2.12 \cos 75$. $F = \mu R$ should now be applied giving $\mu = 2.12 \sin 75/(6 + 2.12 \cos 75) = 0.313$.

Candidate 2 calculated the friction force and the normal reaction incorrectly. $F = \mu R$ was then used. To solve the question it was necessary to resolve in 2 directions. The easiest way to do this was to resolve horizontally and vertically. This gives the friction force and the normal reaction immediately as $F = 2.12 \sin 75$ and $R = 6 + 2.12 \cos 75$. From here $F = \mu R$ is applied to find the coefficient of friction.

Question 3

3 A particle P of mass 0.2 kg is released from rest and falls vertically. At time t s after release P has speed v m s^{-1}. A resisting force of magnitude $0.8v$ N acts on P.

(i) Show that the acceleration of P is $(10 - 4v)$ m s^{-2}. [2]

(ii) Find the value of v when $t = 0.6$. [5]

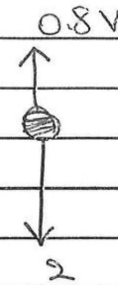
Mark scheme

3 **(i)**	$0.2\,dv/dt = 0.2g - 0.8v$	M1		Use Newton's Second Law, – sign essential
	$a = (dv/dt =)10 - 4v$ AG	A1	[2]	
(ii)	$\int 1/(10-4v)\,dv = \int dt$	M1		Separates variables and attempts to integrate
	$\frac{-1}{4}\ln(10-4v) = t\ (+c)$	A1		
	$[c = -\frac{1}{4}\ln 10]$	M1		Attempts to find the constant or uses the correct limits
	$\frac{-1}{4}\ln(10-4v) = 0.6 - \frac{1}{4}\ln 4$	A1		
	$v = 2.27$	A1	[5]	

Example candidate response – 1

3] $\text{Mass}_p = 0.2\text{ kg}$

(i) $\Sigma F_y = ma$



$2 - 0.8V = 0.2 \text{ acceleration}$ ✓

$\text{acceleration} = \dfrac{2 - 0.8V}{0.2}$ ✓

2

$\boxed{\text{acceleration} = 10 - 4V}$ ∴ Proved.

(ii) $\dfrac{dv}{dt} = 10 - 4V$

~~dv~~ $\displaystyle\int_0^V \frac{1}{10-4V}\,dV = \int_0^{0.6} dt$ ✓

M1.

A0 $\left[\ln|10-4V|\right]_0^V = t\Big]_0^{0.6}$ ✗

M1. $\ln|10-4V| - \ln 10 = 0.6$

A0 $\ln\left|\dfrac{10-4V}{10}\right| = 0.6$

A0 $10 - 4V = 10e^{0.6}$ $\quad V = \dfrac{10 - 10e^{0.6}}{4} =$ ✗

(4) $10 - 10e^{0.6} = -4V$ $\quad |-2.06\text{ m/s}| \rightarrow$ velocity

∴ Speed = 2.06 m/s

Item marks awarded: (i) = 2/2; (ii) = 2/5

Total mark awarded = 4 out of 7

Example candidate response – 2

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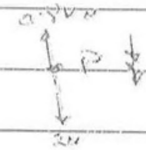
Question 3 — Page number 2

$P = 0.2\,Kg$



i) $F = ma$

$2 - 0.8V = 0.2\left(\frac{dv}{dt}\right) a$ ✓ ✓

2

$a = (10 - 4V)\,ms^{-2}$

ii) $\int_0^{0.6} 10 \mp 4v \; dv$

$[10v \pm 2v^2]_0^{0.6}$

0

$[5.28] - (0) = 5.28\,ms^{-1}$ ✗

2

Item marks awarded: (i) = 2/2; (ii) = 0/5

Total mark awarded = 2 out of 7

Examiner comment – 1 and 2

(i) Candidates 1 and 2 both scored two marks in this part of the question. Both candidates used Newton's second law to give $2 - 0.8v = 0.2a$. This led to the required answer $a = 10 - 4v$ ms^{-2}. This part represents a good example of correct working from both candidates.

(ii) In this part of the question, candidate 1 found the correct integral with the variables separated. However, they then made an error when attempting to integrate. $\ln(10 - 4v)$ was seen instead of $-\frac{1}{4}\ln(10-4v)$. An attempt was then made to find the constant of integration using the correct limits.

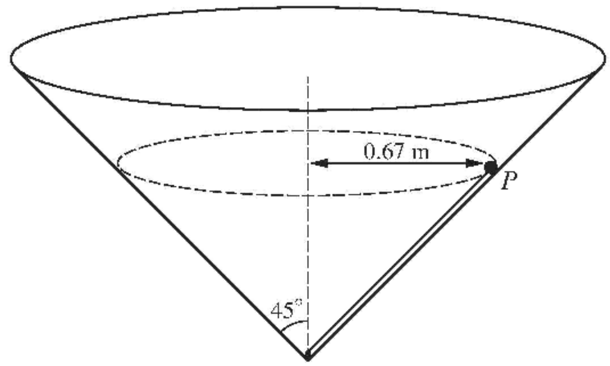
Candidate 2 was unable to separate the variables correctly. No marks were awarded due to the failure to separate correctly.

Separating the variables correctly gives $\int \frac{1}{10-v}\mathrm{d}v = \int \mathrm{d}t$. After integrating the result should be $-\frac{1}{4}\ln(10-4v) = t + c$.

When $t = 0$, $v = 0$, $c = -\frac{1}{4}\ln 10$ and so $-\frac{1}{4}\ln(10-4v) = t - \frac{1}{4}\ln 10$. $t = 0.6$ is now substituted leading to $v = 2.27$.

Question 4

4



A particle P is moving inside a smooth hollow cone which has its vertex downwards and its axis vertical, and whose semi-vertical angle is 45°. A light inextensible string parallel to the surface of the cone connects P to the vertex. P moves with constant angular speed in a horizontal circle of radius 0.67 m (see diagram). The tension in the string is equal to the weight of P. Calculate the angular speed of P. [6]

Mark scheme

4	$R\cos45 - T\cos45 = mg$	M1	Resolves vertically for P
	$R\cos45 = mg + mg\cos45$	A1	May be implied for later work
	$R\sin45 + T\sin45 = m\omega^2 \times 0.67$	M1	Uses Newton's Second Law horizontally for P
		M1	Obtaining an equation in m (and g)
	$mg + mg\cos45 + mg\sin45 = m\omega^2 \times 0.67$	A1	
	$\omega = 6(.00)\ \text{rads}^{-1}$	A1 [6]	

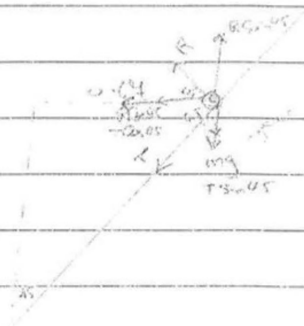
Example candidate response – 1

Question 4 Page number

$T = mg.$

$mg + T\sin 45 = R\sin 45.$ ✓ M1

$mg + mg\sin 45 = R\sin 45.$ ✓ A1.



$R = \dfrac{mg + mg\sin 45}{\sin 45}$

M1.

$m \cdot a = R\cos 45 + T\cos 45.$

$m\omega^2(r) = R\cos 45 + T\cos 45$ ✓ M1

$\cancel{m}\omega^2(0.67) = \dfrac{\cancel{m}g(1+\sin 45)}{\sin 45} + \cancel{m}g\cos 45$ $\quad mg\dfrac{(1+\sin 45)\cos 45}{\sin 45}$

Yes.

A0

$\omega^2(0.67) = (10 + 10\sqrt{2}) + 5\sqrt{2}.$

$\omega^2 = 46.6$ A0.

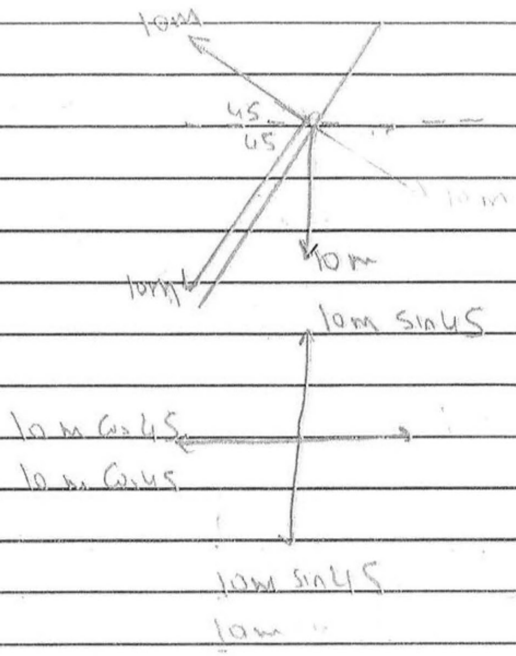
$\omega = 6.83 \text{ rads}^{-1}$ ×

(4).

Total mark awarded = 4 out of 6

Example candidate response – 2

4



$\Sigma F_y = 0$

M1 $10m\sin45 = 10m\sin45 + 10m$

A1.

$\Sigma F_x = 0$

$20\cancel{m}\cos45 = \cancel{m}\,\omega^2 r$

$10\sqrt{2} = \omega^2\, 0.67$

0 $\omega = 4.5$ rad/sec

(2)

Total mark awarded = 2 out of 6

Examiner comment – 1 and 2

In this question, candidate 1 correctly resolved horizontally to get $mg + T\sin 45 = R\sin 45$. The candidate then correctly applied Newton's second law horizontally leading to $m\,\omega^2 \times 0.67 = R\cos 45 + T\cos 45$.
$T = mg$ was then used by the candidate in an attempt to set up an equation in m. However, as seen, this equation was incorrect.

Candidate 2 correctly resolved horizontally and used $T = R = mg$, and Newton's second law was used incorrectly. The equation should contain three terms.

The correct solution should be:
Resolving vertically: $mg + T\sin 45 = R\sin 45$
Newton's second law applied horizontally: $R\cos 45 + T\cos 45 = \mathrm{m} \times 0.67\,\omega^2$
Replacing T by mg , gives $mg + mg\sin 45 = R\sin 45$ and $R\cos 45 + mg\cos 45 = m \times 0.67\,\omega^2$
Eliminating R leads to $mg + mg\sin 45 = m \times 0.67\,\omega^2 - mg\cos 45$. This results in $\omega = 6(.00)$ rads^{-1}.

Question 5

5 A particle P is projected with speed $30\,\text{m s}^{-1}$ at an angle of $60°$ above the horizontal from a point O on horizontal ground. For the instant when the speed of P is $17\,\text{m s}^{-1}$ and increasing,

(i) show that the vertical component of the velocity of P is $8\,\text{m s}^{-1}$ downwards, [2]

(ii) calculate the distance of P from O. [5]

Mark scheme

5	**(i)**	$v^2 = 17^2 - (30\cos 60)^2$	M1		Finds vertical speed
		$v = -8$	A1	[2]	– may be implied by later work
	(ii)	$-8 = 30\sin 60 - gt$	M1		Finds relevant time
		$t = 3.4$	A1		3.398
		$y = [(30\sin 60)^2 - 8^2]/(2g)$ (= 30.55)	B1		Or $y = (30\sin 60) \times 3.4 - g\,3.4^2/2$ (= 30.53)
		$OP^2 = (30\cos 60 \times 3.4)^2 + 30.55^2$	M1		Use of Pythagoras
		$OP = 59.4$ m	A1	[5]	Accept 59.5

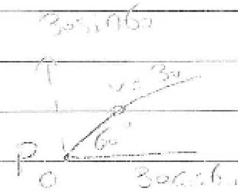
Example candidate response – 1

Question 5 Page number 3

5)

i)

$v = 17 ms^{-1}$



horizontal component is $30\cos 60 = 15 ms^{-1}$

$\therefore \quad 17 = \sqrt{15^2 + ??}$

$17 = \sqrt{15^2 + (-8)^2}$ ✓ ok.

$= 17$

$\therefore$ the vertical compent of velocity is $8 ms^{-1}$ downwards. ✓ 2

ii) $u = 30\sin 60$
$a = -10$
$v = 8$

$-8 = 30\sin 60 - 10t$ M1
$-8 = 15\sqrt{3} - 10t$ ✓ A1.
$t = 3.3980762\,11$

$S = 30.55m$ ✓ B1.

$\therefore \quad u = 30\sin 60$
$a = -10$
$t = 3.398076211$

$\therefore S = ut + \frac{1}{2}at^2$

$S = 30\sin 60(3.398076211) - 5(3.398076211)^2$ M0 A0

5

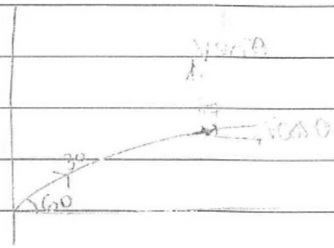
Item marks awarded: (i) = 2/2; (ii) = 3/5

Total mark awarded = 5 out of 7

Example candidate response – 2

5) i) HL → $30\cos 60 = 17\cos\theta$

$\cos\theta = \frac{30\cos 60}{17}$



$\cos\theta = \frac{15}{17}$

$\sin\theta = \frac{8}{17}$

M1 VL → $17\sin\theta =$

A0 $V_{VL} \to 17\left(\frac{8}{17}\right) = 8$ ✓ ^ Div ✓:

ii) VL → $v = u + at$

$8 = 30\sin 60 - 10t$

M0 $10t = 30\sin 60 - 8$

$t = 1.798$ ✗

A0 $v^2 = u^2 + 2as$

$8^2 = (30\sin 60)^2 - 20S$

$20S = (30\sin 60)^2 - 8^2$ ✓

B1 $S = 30.55$

HL → $S = \frac{d}{t}$

M1 $15 = \frac{d}{1.798}$ $d = 26.97$

A0 Distance $= \sqrt{30.55^2 + 26.97^2} = 40.8$ m ✗

3

Item marks awarded: (i) = 1/2; (ii) = 2/5

Total mark awarded = 3 out of 7

Examiner comment – 1 and 2

(i) In this part of the question, candidate 1 scored both marks. The horizontal component of the velocity, $30\cos 60 = 15$, was correctly found. The application of Pythagoras's theorem resulted in the vertical component of the velocity being $8\,\text{ms}^{-1}$ downwards, in line with the mark scheme,

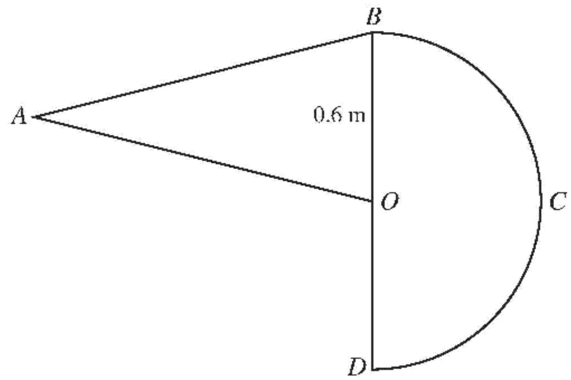
Candidate 2 also correctly calculated the vertical component of the velocity, but did not specify the direction. This meant that the second mark was not awarded for this working.

(ii) Again, in this part candidate 1 found the correct time to travel to P by using $-8 = 30\sin 60 - gt$. This led the candidate to $t = 3.4$. The vertical distance of P above the horizontal was found correctly by applying $s = ut + \frac{1}{2}at^2$. This gives the correct vertical distance as 30.55 m. No further work by the candidate was seen. The horizontal distance is $30\cos 60 \times 3.4 = 51$ m. Pythagoras's theorem results in $OP = \sqrt{(30.55^2 + 51^2)} = 59.4\,\text{m}$

Candidate 2 used $8 = 30\sin 60 - gt$. This equation gives the time when the particle is travelling upwards and not downwards. The candidate should have used $-8 = 30\sin 60 - gt$. This correct equation gives $t = 3.4$. The correct vertical distance was to be found by using $82 = (30\sin 60)2 - 2gs$. The vertical distance is 30.55 m. An incorrect horizontal distance was calculated and an attempt at Pythagoras's theorem was seen.

Question 6

6



A uniform lamina $OABCD$ consists of a semicircle BCD with centre O and radius 0.6 m and an isosceles triangle OAB, joined along OB (see diagram). The triangle has area $0.36\,\text{m}^2$ and $AB = AO$.

(i) Show that the centre of mass of the lamina lies on OB. [4]

(ii) Calculate the distance of the centre of mass of the lamina from O. [4]

Mark scheme

6	**(i)**	Height of triangle = $0.36/0.3 (= 1.2\text{ m})$	B1		
		Semi-circle C of M = $2 \times 0.6/(3\pi/2)$	B1		Centre of mass lamina from BOD
		$0.36 \times (1.2/3) = \pi \times 0.6^2/2 \times 2 \times 0.6/(3\pi/2)$	M1		Equating moments idea
		$0.144 = 0.144$	A1	[4]	Evidence of checking equality
OR		$0.36 \times (1.2/3) - \pi \times 0.6^2/2 \times 2 \times 0.6/(3\pi/2)$			
		= distance × total area	M1		Table of moments idea
		Distance = 0	A1		
	(ii)	0.36×0.3	A1		Correct sum of parts
		$= (0.36 + \pi\, 0.6^2/2) \times OG$	A1		Correct moment of whole
		$OG = 0.117\text{ m}$	A1	[4]	

Example candidate response – 1

(6)

i) For the cg to be on OB ~~M~~ M of Δ = M of $\bigtriangleup$ about O

Cg of $\bigtriangleup = \frac{2(0.6)\sin(90)}{3\pi/2} = 0.25465$ from O

Cg of $\Delta = \frac{1}{3}h$ from Base $\frac{1}{3}(1.2) = 0.4$

$\frac{1}{2} \times 0.6 \times h = 0.36 \quad h = 1.2$

$0.4(0.36) = 0.25465(0.5 \times 0.6^2 \times \pi)$

$0.4(0.36) = \frac{2(0.6)\sin 90}{3\pi/2} \times (0.5 \times 0.6^2 \times \pi)$

$0.144 = \cancel{0.0??} \quad 0.144$ 4.

ii) $\bar{y} = \frac{0.3(0.36)}{0.36 + 9\pi/100} = 0.168$ above O M1. A1.

$\frac{1}{2}\pi \times 0.6^2 = 0.18\pi = \frac{8\pi}{100}$ A0 A0.

(6)

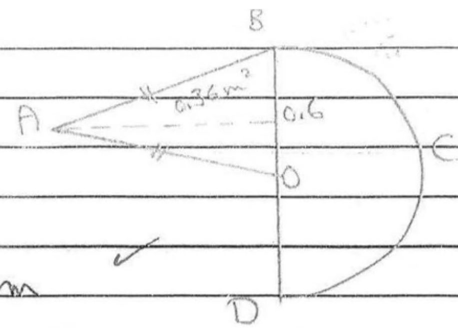
Item marks awarded: (i) = 4/4; (ii) = 2/4

Total mark awarded = 6 out of 8

Example candidate response – 2

6 i) height of Δ:

area $= \frac{1}{2} b \times h$



$0.36 = 0.6 \times \frac{1}{2} \times h$

$\therefore h = 1.2\ m$ ✓ B1.

Shape	Area	CG from OB
Triangle	0.36	0.4 ✓
Semi-circle	0.18π	$\frac{2(0.6)\sin\frac{\pi}{2}}{1.5\pi}$
lamina	$0.36+0.18\pi$	$\bar{x}$

B1. M1.

$0.36 \times 0.4 + 0.18\pi \times \frac{4}{5\pi} = \bar{x}(0.36+0.18\pi)$ A0.

~~$0.288 = \bar{x}(0.36+0.18\pi)$~~

for the CG to lie on OB, $\bar{x}$ must be $=0$

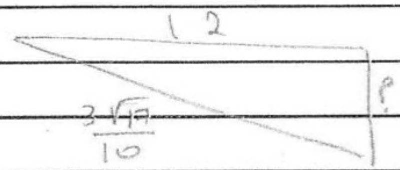
Example candidate response – 2, continued

ii) $\Sigma M_{A} = 0$

$(0.36)(0.8) + (0.18\pi)\left(1.2 + \frac{1.2}{1.5\pi}\right) = \bar{x}\,(0.36 + 0.18\pi)$ ✓

$\bar{x} = 1.2$

$\overline{AO} = \sqrt{(1.2)^2 + (0.3)^2} = 1.24 \quad \left(= \frac{3\sqrt{17}}{10}\right)$



Distance $= \sqrt{\left(\frac{3\sqrt{17}}{10}\right)^2 + (1.2)^2}$

0 $= 1.72$ m ✗

3

Item marks awarded: (i) = 3/4; (ii) = 0/4

Total mark awarded = 3 out of 8

Examiner comment – 1 and 2

(i) In this part of the question, candidate 1 scored all four marks and candidate 2 the first three marks. Both candidates correctly found the centres of mass of the semi-circular lamina and the triangular lamina as 0.25465 m and 0.4 m respectively from BD.

Candidate 1 then established the correct moment equation and went on to show that the centre of mass of the whole lamina was on OB.

Candidate 2 was not awarded the fourth mark because a sign error occurred in the moment equation.

(ii) Candidate 1 took moments about O with all the correct terms present. An error appeared when finding the area of the semi-circular lamina. The area should have been $\frac{1}{2}\pi \times 0.6^2 = \frac{18}{100}\pi$ not $\frac{9}{100}\pi$.

Candidate 2 tried to take moments about A rather than O, so no marks were awarded.

Taking moments about O gives $0.3 \times 0.36 = 0.36 + \pi \times \frac{0.6^2/2}{2}y$, where y is the required distance.

Question 7

7 A light elastic string has natural length 3 m and modulus of elasticity 45 N. A particle P of weight 6 N is attached to the mid-point of the string. The ends of the string are attached to fixed points A and B which lie in the same vertical line with A above B and $AB = 4$ m. The particle P is released from rest at the point 1.5 m vertically below A.

(i) Calculate the distance P moves after its release before first coming to instantaneous rest at a point vertically above B. (You may assume that at this point the part of the string joining P to B is slack.) [4]

(ii) Show that the greatest speed of P occurs when it is 2.1 m below A, and calculate this greatest speed. [5]

(iii) Calculate the greatest magnitude of the acceleration of P. [3]

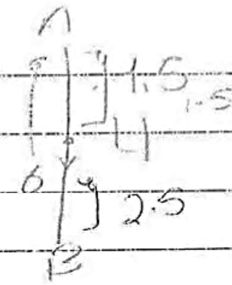
Mark scheme

7 (i)		M1	Energy conservation, no KE, 2 EE terms
	$45 \times 1^2/(2 \times 1.5) + 0.6\,gh = 45\,h^2/(2 \times 1.5)$	A1	
	$5h^2 - 2h - 5 = 0$	M1	Simplifies, tries to solve a 3 term quadratic equation
	$h = 1.22$ m	A1 [4]	
(ii)	$45e/1.5 = 45(1 - e)/1.5 + 6$	M1	Finds equilibrium position ($e = 0.6$)
	$AP = (1.5 + 0.6) = 2.1$ AG	A1	
	$0.6\,v^2/2 = 0.6\,g \times 0.6 + 45\,(1)^2/(2 \times 1.5)$ $- 4.5(0.6)^2/(2 \times 1.5) - 45(0.4)^2/(2 \times 1.5)$	M1 A1	Energy conservation with KE/PE/EE terms
	$v = 6$ ms^{-1}	A1 [5]	
(iii)	$0.6\,a = \pm\,(0.6g + 45 \times 1/1.5)$	M1*	Top $a = \pm\,60$ ms^{-2}
	$0.6\,a = \pm\,(0.6g - 45 \times 1.22/1.5)$	M1*	Bottom $a = \pm\,51$ ms^{-2}
	$\lvert a \rvert = 60$ ms^{-2}	A**1 [3]	Needs acceleration at both extreme positions considered.

Example candidate response – 1

Question 7 Page number 3

$l = 3 \quad T = 45$



(ii) energies at P = energies at P'

$$\frac{45(1)^2}{2(1.5)} + 0.6(g)(2.5) = \frac{45(x^2)}{2(1.5)} +$$

$$0.6(g)(2.5 - x)$$

$$\frac{45}{3} + 15 = 15x^2 + 15 - 6x$$

$$15x^2 - 6x = 15$$

$$15x^2 - 6x - 15 = 0 \qquad x = 1.22$$

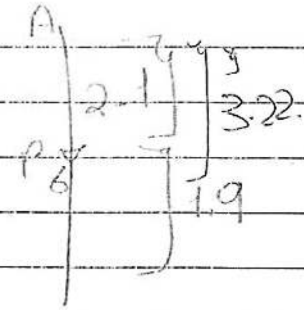
4. $AP = 3.22 \qquad AP = 1.5$

distance = 1.72 m. isw.

Example candidate response – 1, continued

ii)

$T_1 = W + T_2$



$\frac{45(\cancel{x}-1.5)}{\cancel{2}(1.5)} = \frac{45(4-x-1.5)}{1.5} + 6$

$\cancel{30x - 45 = \frac{6}{5} + 6}$

$\frac{45(x-1.5)}{1.5} = \frac{45(4-x-1.5)}{1.5} + 6$ ✓

$30x - 45 = 75 - 30x + 6$

$60x = 75 + 6 + 45$

$x = 2.1$ m ✓ 2

$\frac{45(0.6)^2}{2(1.5)} + \frac{45(0.4)^2}{2(1.5)} + 0.6(10)(1.12)$ ✗ M0

$+ \frac{1}{2}(0.6)v^2 = \frac{45(1.72)^2}{2(1.5)}$ ✗ A0

$\frac{27}{5} + \frac{12}{5} + 6.72 + 0.3v^2 = 44.376$ A0

$v = 10$ m/s ✗

Example candidate response – 1, continued

iii) $0.6(a) = 6 + T$

1.5

$0.6a = 6 + \frac{45(1)}{1.5}$

$0.6a = 36$

M1 greatest acceleration $= 60\ m/s^2$.

M0 A.

A0

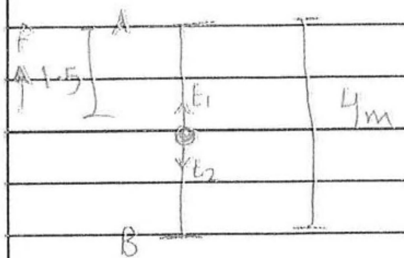
7

Item marks awarded: (i) = 4/4; (ii) = 2/5; (iii) = 1/3

Total mark awarded = 7 out of 12

Example candidate response – 2

7 $l_n = 3 \quad \lambda = 45$



$t_1 = t_2 + 6$

$\frac{-45(0.5)}{\cancel{3}\,1.5} = \frac{45(0.5)}{\cancel{3}\,1.5} + 6$

$\frac{45(x-0.5)^2}{2\times1.5} = 6(x-0.5) + \frac{45(1)^2}{2\times1.5}$

$15(x^2 - x + 0.25) = 6x - 3 + 15$

$15x^2 - 15x + 3.75 = 6x + 12$

$\cancel{15x^2 \ 21x}$ $15x^2 - 21x - 8.25 = 0$

$x = 1.7198, \qquad x = -0.3198$ rejected

4 ∴ distance moved $1.7198 - 0.5$
$\boxed{1.22}$

(ii) $\frac{45(x+0.5)}{\cancel{2}\,1.5} = \frac{45(x-\cancel{0.5})}{1.5} + 6$

$\cancel{30x + 15 = 30x + 6}$

0 $\frac{45(2.1-1.5)^2}{2\times1.5} = 6\times2.1 + \frac{1}{2}\times0.6\times v^2 + \cancel{\frac{45(1.22)^2}{2\times1.5}}$

$v^2 =$

(4)

Item marks awarded: (i) = 4/4; (ii) = 0/5; (iii) = 0/3

Total mark awarded = 4 out of 12

Examiner comment – 1 and 2

(i) In this part of the question, both candidate 1 and 2 scored all four marks. The correct energy equation was set up by both candidates, giving the distance moved as 1.22 m.

(ii) In this part, candidate 1 managed to show that the greatest speed occurred when P was at a distance of 2.1 m below A. This occurs when the acceleration is zero and so $T_A = W + T_B$. An incorrect attempt was made at an energy equation. This equation should be $\frac{0.6v^2}{2} = 0.6g \times 0.6 + \frac{45(1)^2}{(2 \times 1.5)} - \frac{45(0.6)^2}{(2 \times 1.5)} - \frac{45(0.4)^2}{(2 \times 1.5)}$.

This correct equation leads to $v = 6\,\text{ms}^{-1}$.

Candidate 2 scored no marks in this part of the question.

To find the equilibrium position it is necessary to use $T_A = T_B + 6$ with e as the extension of the top section of the string. This gives $\frac{45e}{1.5} = \frac{45(1-e)}{1.5+6}$, $e = 0.6$ and so the distance below A is 1.5 + 0.6 = 2.1 m. An energy equation at this equilibrium position is now needed. This equation can be seen above.

(iii) In the final part of the question, candidate 1 found the acceleration at the top, but did not compare it with that at the bottom.

Candidate 2 did not attempt this part of the question, so no marks were awarded.

The correct solution is as follows:

Newton's second law should be used at both the top and bottom of the motion. The greater of the accelerations should then be taken. At the top, $0.6a = 0.6g + 45 \times \frac{1.22}{1.5}, a = 60$.

At the bottom. $0.6a = 0.6g - 45 \times \frac{1.22}{1.5}, a = -51$. Greatest magnitude of the acceleration is 60 ms^{-2}.

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